Capital Budgeting, Innovation, and Free-Riding

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Abstract

Firms often underinvest in implementing ideas, decreasing the probability that these ideas turn into profitable innovations. In this article, I rationalize why firms invest less than seemingly optimal levels into implementing ideas. I consider a setting where an employee and employer sequentially engage in ideation. The employer obtains a profitable innovation if an idea is generated and successfully implemented. I show that greater investment affects the employer's payoff through two conflicting channels. First, it increases the likelihood of successfully implementing an idea, leading to increased innovation. Second, it increases the employee's incentive to free-ride off the employer's ideation effort, decreasing idea generation and innovation. Thus, relative to a setting with only idea implementation, the employer optimally *underinvests* in a setting where idea generation is also considered in a bid to incentivize the employee to ideate. Among other things, I use the model to generate insights regarding (1) the relationship between investment and the generation of ideas within firms, (2) the effect of employee creativity on investment, and (3) how employee creativity influences the employee's generation of ideas.

Keywords. Ideation, implementation, investment, money-burning, free-riding, innovation, commitment-problem.

1 Introduction

An oft-cited reason why innovations fail to materialize within firms is that innovative ideas are poorly implemented due to 'underfunding.'[1] Among other things, underfunded ideas have to make do with lower-quality inputs or undergo less testing and prototyping, making them unlikely to turn into profitable products or services. But why would firms self-sabotage by seemingly underinvesting in implementing innovative ideas relative to what is dictated by the NPV rule? The answer to this question is not obvious.

In this article, I outline a dual role of the firm's investment choice. Greater investment obviously leads to better implementation of ideas. However, greater investment reduces employees' incentive to generate ideas and is, therefore, undesirable from an innovation perspective. Thus, underinvestment is puzzling when viewed purely from the lens of implementation. However, when the generation and implementation of ideas are jointly considered, underinvestment arises as an equilibrium outcome. The main contribution of this article is to show that firms underinvest in implementing ideas (relative to what is dictated by the NPV rule) to spur employees to ideate.

To elaborate, I consider a setting with an employer ('she') who wants to generate profits by introducing an innovative product or service. The process of innovation comprises two steps: 'ideation' and 'implementation' (Levitt, 2002; Katila and Shane, 2005; Gundry et al., 2016). Any profitable innovation first materializes as an idea. However, the idea is profitable only if it is implemented properly. For instance, the idea can be a revolutionary new medical treatment. This revolutionary new medical treatment might be theoretically exciting. Still, it is of no value to the firm if it cannot be administered to patients.[2]

The employer hires an employee ('he') to engage in (personally costly) ideation. If the employee generates an idea, the employer implements the idea. If the employee is unsuccessful, the employer engages in ideation. If she succeeds at generating an idea, she implements it; otherwise, the game ends. Both players have an intrinsic

[1] A survey by EY found that 42% executives consider budget constraints to be a barrier to 'activation' (or implementation) of innovative ideas. Another example of conventional wisdom bemoaning budget issues is as follows: '*One of the most common reasons of innovation failure is a lack of budget or money invested in innovative approaches.*'

[2] Such was the case with the Theranos 'Edison' blood testing machines. The idea behind these machines was that patients would only need to give a single drop of blood to conduct a panel of tests. While the idea was theoretically exciting, it ultimately failed because the machine could never be developed (Carreyrou, 2019).

desire to generate an idea: the employer obtains a profitable innovation if the idea is successfully implemented, and the employee obtains a private benefit which captures that an idea moves on to the implementation stage where, regardless of the outcome, he can enjoy perks or acquire valuable skills.

Since both the employer and employer benefit when a project is implemented, the employer's and the employee's ideation efforts are substitutes – as long as one of them generates an idea, both are better off. This substitutability incentivizes the employee to free-ride. He anticipates that undersupplying ideation effort is relatively inexpensive. He can under-ideate, not generate an idea, and simply report that he failed at ideating. The employer will then ideate, which, of course, benefits her but also 'bails out' the employee if she is successful at generating an idea.

At the start of the game, the employer chooses an investment level subject to a cost. This investment influences the probability that an idea is successfully implemented. Specifically, a larger investment implies that the idea can use higher quality inputs or undergo extensive prototyping, increasing the probability that the idea will turn into an innovative product or service.

In characterizing the employer's optimal investment level, I begin with a benchmark setting where ideation is not required. Thus, the employer has an idea on hand and only needs to implement it. In this benchmark case, an increase in investment affects the employer's payoff via two channels. First, it increases the probability that the idea is successfully implemented; I call this the 'implementation effect' of greater investment. The implementation effect captures a standard argument favoring greater investment from an innovation perspective and increases the employer's payoff. Second, greater investment is costly and decreases the employer's payoff. The employer trades off these two channels to determine the investment benchmark level.

When ideation is added to the benchmark, an increase in investment also affects the employer's payoff via a previously undocumented channel termed the 'free-rider effect.' The logic for the free-rider effect is as follows. An increase in investment increases the employer's expected value from generating an idea since any idea is now more likely to be successfully implemented, prompting her to increase her ideation. Anticipating this increase in the employer's ideation, the employee reduces his ideation. He rationalizes that even if he undersupplies ideation effort and fails to generate an idea, the employer will likely bail him out due to her increased ideation. Thus, the free-rider effect captures that greater investment *decreases* the employer's payoff by spurring the employee to free-ride off the employer's ideation. The employer trades off the positive implementation effect against the negative free-rider effect and the cost of greater investment in determining her equilibrium investment policy.

Comparing the investment levels shows that the employer *underinvests* in the main setting relative to the benchmark. This result follows due to the free-rider effect; the benchmark and main settings are similar except for the free-rider effect, which is negative in the main setting. Intuitively, greater investment in both settings increases the employer's payoff by increasing the likelihood of successfully implementing an idea. However, the employer is also incentivized to reduce her investment in the main setting. Since a reduction in investment destroys the expected value of an idea (idea implementation is less likely to succeed with a smaller investment), the employer's *ex-post* ideation decreases. The employee anticipates this reduction in the employer's ideation and responds by increasing his ideation because the decrease in the employer's ideation renders free-riding less valuable. This increase in the employee's ideation increases the employer's payoff.[3]

The underinvestment result can be reinterpreted in terms of a 'commitment problem.' Suppose the timeline of the game is as follows: the employer chooses her investment policy at date 0, the employee ideates at date 1, the employer ideates at date 2 (if the employee's ideation is unsuccessful), and an idea, if generated, is implemented at date 3. Then, if the employer could, he would like to commit to a lower level of ideation at date 0 relative to what she chooses at date 2. The logic is as follows. The employer's ideation at date 2 only depends on the expected value of an idea; the employee's ideation is irrelevant because it is sunk (and unsuccessful) at date 1 by the time the employer ideates at date 2. However, at date 0, the employer also factors in how her ideation affects the employee's incentive to ideate at date 1. She rationalizes that ideating at the date 2 optimal level induces the employee to excessively free-ride, prompting her to prefer a relatively lower level of ideation at date 0. However, she cannot credibly commit to lesser ideation at date 0, giving rise to a commitment problem.

Instead, the employer uses her investment level at date 0 to circumvent her commitment problem. A decrease in investment reduces her expected payoff from an idea (since any idea is less likely to be successfully implemented). In turn, she ideates less at date 2. That is, a smaller investment at date 0 *commits* the employer to lesser ideation at date 2, which is valuable to resolve her commitment problem.

[3] An envelope result implies that since the employer's ideation is chosen optimally, any effect of the investment on her ideation does not have a bearing on her payoff.

Recall that in the benchmark setting, there is no ideation and, thus, no commitment problem. Since the benchmark and main settings are similar otherwise, the commitment problem introduced by ideation prompts the employer to choose a smaller investment level in the main setting relative to the benchmark.

1.1 Contribution and Background Literature

Prior literature has documented the free-rider problem in the context of ideation (Williams et al., 1981; Harkins and Petty, 1982; Kerr and Bruun, 1983; Toubia, 2006). A common theme in this literature is that if employee performance can be directly measured and rewarded (instead of being rewarded based on a team measure), then the free-riding problem in ideation is alleviated. For instance, Toubia (2006) shows that financial contracts that reward an employee for his contribution to a firm's overall innovation can reduce free-riding and bolster ideation. The main contribution of my model is to show that firms can strategically choose their capital allocations to further alleviate the free-rider problem even when optimal incentive contracting (à la Toubia (2006)) is considered. In doing so, I provide a novel explanation for why firms seemingly underinvest in implementing ideas relative to what would be dictated by the NPV rule.

More generally, my setting is related to the literature that examines capital budgeting in the presence of information asymmetry (e.g. Antle and Eppen (1985), Harris and Raviv (1996), Bernardo et al. (2001), Kim (2006), and Bernardo et al. (2009)). Similar to this literature, I examine how capital budgeting decisions within a firm depend on the employee's moral hazard problem in generating ideas (ideation). The closest paper in this literature is Bernardo et al. (2009), who study a setting where an employee has to ideate and subsequently report the quality of her idea. They show that the investment policy balances ideation incentives across truthful reporting incentives. Similar to their setting, the employer in my setting chooses her investment policy to provide the employee with ideation incentives. However, the employer's investment affects the employee's incentive to ideate in my setting due to a free-riding concern, which is absent in their setting.

From a modeling perspective, my model builds on the seminal Aghion and Tirole (1997) model. In their setting, an employer and an employee simultaneously search for a project (similar to ideation in my setting). However, there is no free-riding in their setting. I build on their setting by introducing a capital budgeting decision by the employer.

As discussed before, the underinvestment result in my setting can be interpreted through the lens of a commitment problem. Prior literature has also studied such commitment problems in several different contexts (e.g. Crémer (1995), Burkart et al. (1997), Aghion and Tirole (1997), Arya and Glover (2014), Laux (2017)). A common theme in this literature is that a principal undertakes a certain *ex-post* action which is suboptimal from an *ex-ante* perspective. For example, Burkart et al. (1997) study a setting where the shareholders monitor a CEO. After the CEO has exerted effort into identifying projects to pursue, the shareholders intensely monitor the CEO to discipline his project choice. However, before the CEO has exerted effort, the shareholders would *like to* commit to a lower level of monitoring. A lower level of monitoring implies that the CEO can choose his 'pet project,' which gives him strong incentives to identify projects.

My article also contributes to the literature by generating several novel implications, such as the relationship between (1) employee creativity and investment, (2) investment and the generation of ideas, and (3) employee creativity and the generation of ideas (amongst others).

2 Model Setup

Players There are two risk-neutral players in the game – an employer and an employee.

Timeline At Date 0, the employer commits to a level of investment, I. At Date 1, the employee engages in ideation. At Date 2, if the employee fails to come up with an idea for an innovation, the employer engages in ideation.[4] If the employee or employer succeeds at ideating, the employer implements the idea at Date 3. The success of the implementation depends on the employer's investment. At Date 4, payoffs are realized. The timeline is depicted in Figure 1.

Innovation The process of innovation can be broken down into two main components: ideation and implementation. 'Ideation' is the creative part of innovation where an idea is generated. However, the innovation only materializes when this idea is successfully executed or 'implemented.'

[4]As in Aghion and Tirole (1997), the main result is insensitive to whether the employee and employer move sequentially or simultaneously. For expositional simplicity, I focus on the case where they move sequentially.

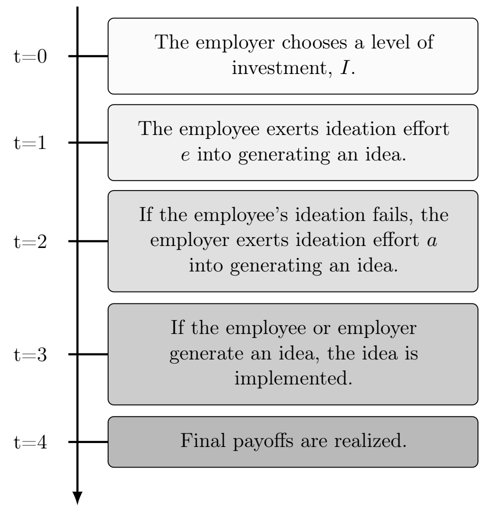

Figure 1: The Timing of the Game

Ideation The employee's ideation is parameterized by e. With probability e, the employee's ideation is successful, and he generates an idea. With probability $(1-e)$, his ideation fails and no idea is generated. His cost of ideating is $\frac{le^2}{2}$ where $l > 0$ can be interpreted as a measure of the employee's 'creativity.' A larger (smaller) l proxies for lower (greater) creativity. If the employee's ideation fails, the employer engages in ideation; her ideation is parameterized by a. Like the employee, she generates an idea with probability a and fails with probability $(1-a)$. Her cost of ideating is $\frac{ce^2}{2}$ where $c > 0$. If the employee or the employer fails to generate an idea, the game ends. On the other hand, when an idea is generated, it moves on to the implementation phase.

Implementation The employer commits *ex-ante* to a level of investment, I, at a cost of $G(I) = \frac{gI^2}{2}$ where $g > 0$.[5] This level of investment influences the likelihood that the idea is successfully implemented. Specifically, I assume that with probability I, implementation succeeds, and with probability $(1-I)$, implementation fails. This technology captures the idea that a better-funded idea can use higher-quality inputs or go through more prototyping which increases the probability that it's successfully implemented. If the implementation is successful, the employer obtains an innovation

[5]The assumption that the employer commits to a level of investment *ex-ante* has been used extensively in prior literature (e.g. Bernardo et al. (2001), Bernardo et al. (2004), and Bernardo et al. (2009)) and can be rationalized as the result of reputational concerns in a future (unmodeled) investment game.

worth X; if it is unsuccessful, the employer obtains zero.

Employee Preferences Following prior papers in the information acquisition literature, I take an incomplete contracting approach where none of the employee's actions or outcomes are contractible.[6] Instead, I assume that if ideation is successful, the employee obtains a private benefit $\psi > 0$.[7] This private benefit captures the notion that an idea moves on to the implementation stage, where, regardless of the implementation outcome, the employee can enjoy perks or acquire skills that are valuable in a future job market.

***Ex-ante* Payoffs**

Employer's Payoff

$$V := (e + (1 - e)a)IX - (1 - e)\frac{ca^2}{2} - G(I). \tag{1}$$

The first term is the employer's expected payoff from generating an innovation. With probability $(e+(1-e)a)$, an idea is generated. If the idea is successfully implemented (with probability I), it creates an innovation worth X. The second term is her cost of ideating. The final term is her cost of investment.

Employee's Payoff

$$\Upsilon := (e + (1 - e)a)\psi - \frac{le^2}{2}. \tag{2}$$

The first term in Υ is the employee's expected payoff when an idea is generated. With probability $(e + (1 - e)a)$, the employee or the employer succeeds at ideating, yielding the employee his private benefit, ψ. The second term in Υ is the employee's cost of ideation.

Assumption 1. *I assume:*

[6] See for instance: Aghion and Tirole (1997), Burkart et al. (1997), Baker et al. (1999), and Mello and Ruckes (2006).

[7] This assumption is for simplicity. In a later section, I consider a setting where the employer also provides the employee with an incentive contract to ideate and show that the main result is robust to the introduction of incentive contracting.

1. $\psi < l$, *and*

2. $X < \min\{c, g\}$.

These parametric restrictions ensure that the equilibrium level of investment is interior and the employer's and employee's ideation efforts are interior for any level of investment.

3 Ideation Efforts and the Free-Rider Problem

I solve the game using backward induction. Thus, I first solve for the employer's and then for the employee's ideation levels. Finally, I conclude this section by discussing the free-rider problem in my setting.

The employer's problem at Date 2 is:

$$\max_{a} \; a(IX) - \frac{ca^2}{2}. \tag{3}$$

The first term is her payoff from innovating, given that the employee failed to ideate. The employer finds an idea with probability a; the idea yields her X if the implementation is successful with probability I. The second term is her expected cost of engaging in ideation.

The first-order condition is:

$$IX - ca = 0. \tag{4}$$

The first term on the left reflects that an increase in a increases the probability that the employer generates an idea worth IX in expectation. The second term reflects that greater ideation is costly. I denote the a that solves the first-order condition as a'.

Lemma 1. *The employer's ideation increases in her investment. Formally,*

$$\frac{\mathrm{d}a'}{\mathrm{d}I} > 0. \tag{5}$$

Proof. Follows from Equation (4). ■

The employer finds ideation valuable only to the extent that the generated idea will be successfully implemented in the future. For example, a theoretically revolutionary medical treatment is only profitable if it can be practically administered to patients. Greater investment increases the likelihood that an idea will be successfully implemented; in turn, the employer is more motivated to generate ideas.

The employee's problem is to maximize Υ with respect to e, anticipating a':

$$\max_{e} \ \Upsilon(a') := (e + (1-e)a')\psi - \frac{le^2}{2}. \tag{6}$$

The first-order condition to his problem is:

$$(1-a')\psi - le = 0. \tag{7}$$

The first term on the left reflects that an increase in the employee's ideation increases the probability that an idea is generated, which, in turn, yields him his private benefit. The second term is his marginal cost of greater ideation. I denote the e that solves the first-order condition as e'.

Proposition 1 (Free-Rider Problem)**.** *As the employer's ideation increases (for exogenous reasons), the employee's ideation decreases. Formally,*

$$\frac{\mathrm{d}e'}{\mathrm{d}a'} < 0. \tag{8}$$

Proof. Follows from Equation (7). ■

The employee's incentive to ideate stems from a desire to avoid a situation where he does not have a private benefit to consume. When the employer ideates, the employee knows that failure to generate an idea is not too costly – there is a non-zero probability that the employer will generate an idea, allowing the employee to consume a private benefit despite failure. Consequently, the employee's downside of failure is smaller, prompting him to reduce his ideation. An increase in the employer's ideation further decreases the employee's downside of failure and, thus, his ideation.

4 Investment and Idea Generation

How does the employer's investment affect the probability that an idea is generated within the firm? To study this question, I define the (total) probability of successful ideation as:

$$\iota(a', e') = e' + (1 - e')a'. \tag{9}$$

To study the effect of investment on the probability of successful ideation, I differentiate $\iota(e', a', I)$, with respect to the employer's investment, I, which gives me (with a slight abuse of notation):

$$\frac{\mathrm{d}\iota(e', a', I)}{\mathrm{d}I} = \underbrace{\frac{\partial \iota}{\partial a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I}}_{>0} + \underbrace{\frac{\partial \iota}{\partial e'} \cdot \frac{\mathrm{d}e'}{\mathrm{d}a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I}}_{<0}. \tag{10}$$

The first term in Equation (10) captures the effect of greater investment on the probability of successful ideation through the employer's ideation. Lemma 1 shows that greater investment motivates the employer to ideate and, thus, increases the probability of successful ideation.

The second term in Equation (10) reflects the effect of greater investment on the probability of successful ideation through the employee's ideation. Combining Lemma 1 with Proposition 1 shows that greater investment demotivates the employee to ideate. Free-riding is at the heart of why greater investment demotivates the employee. As Lemma 1 establishes, greater investment spurs the employer to ideate. Thus, upon observing an increase in investment, the employee rationalizes that he will enjoy his private benefit even if he does not identify a project since the employer is likely to 'bail him out' by generating an idea. In turn, he reduces e', adversely impacting the probability of successful ideation.

On the whole, then, the sign of Equation (10) is *ex-ante* ambiguous. The employee's creativity, l, sheds more light on the sign of Equation (10). When l is large ($l > \hat{l}$), the employee's ideation is costly, prompting the employee to ideate less. In turn, any free-riding induced by an increase in investment is small. Stated differently, the downside of greater investment on the probability of successful ideation, as depicted by the second term in Equation (10), is small for a large l. Consequently, the upside of an increase in investment, as depicted by the first term in Equation (10), dominates, and the probability of successful ideation increases as investment increases. This intuition reverses when l is small ($l < \hat{l}$). In this case, the employee's ideation is large, and the free-riding induced by an increase in investment is consequential. Formally, the negative second term in Equation (10) dominates the positive first term, and the probability of successful innovation decreases as investment increases.

The following proposition summarizes.

Proposition 2 (Investment and Idea Generation)**.** *The probability of successful ideation, $\iota(a', e')$, decreases in investment iff the employee is sufficiently creative, $l < \bar{l}$. The threshold $\bar{l}$ is defined in the appendix.*

Proof. See appendix. ■

5 Optimal Level of Investment

In this section, I characterize the employer's optimal level of investment. The main result of this article is that employers 'underinvest' to spur employees to ideate. To highlight this role of the employer's investment on ideation, I begin with a benchmark where ideation is not required. Then, I consider the main setting with ideation. I conclude the section by showing that the employer's investment in the main setting is lower than in the benchmark.

5.1 No-Ideation Benchmark

For the moment, I suppose that the employer has an idea and only needs to implement it. In this case, the employer simply needs to choose an investment level that maximizes the likelihood of successful implementation, subject to the cost of sourcing the investment. Formally, the employer's problem is:

$$\max_{I} \; V_{BM} := IX - G(I). \tag{11}$$

The first-order condition to this problem (with a slight abuse of notation) is:

$$X - \frac{\mathrm{d}G}{\mathrm{d}I} = 0. \tag{12}$$

The first term captures that an increase in investment increases the likelihood that project implementation succeeds. I call the first term the '*implementation effect*' of greater investment on the employer's payoff. The second term captures the marginal cost of greater investment.

Lemma 2 (Equilibrium in the benchmark)**.** *The equilibrium level of investment in the benchmark is the 'I' that solves Equation (12) and is denoted by I_{BM}.*

Proof. Follows from Equation (12). ■

5.2 Main Setting

I return to the full model now. Here, in determining her level of investment, the employer needs to consider both ideation and implementation. Formally, her problem is to maximize her payoff with respect to her investment, anticipating the search efforts:

$$\max_{I} \; V(I, e', a', G) := (e' + (1 - e')a')IX - (1 - e')\frac{c(a')^2}{2} - G(I). \tag{13}$$

The first-order condition to this problem is:

$$\underbrace{\frac{\partial V(I, e', a', G(I))}{\partial I}}_{\text{Implementation Effect}} + \underbrace{\frac{\partial V(I, e', a')}{\partial e'} \cdot \frac{\mathrm{d}e'}{\mathrm{d}a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I}}_{\text{Free-Rider Effect}} + \underbrace{\frac{\partial V(I, e', a')}{\partial a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I}}_{= 0\text{: Envelope Theorem}} - \frac{\mathrm{d}G}{\mathrm{d}I} = 0. \tag{14}$$

The implementation effect has been discussed before. Thus, I focus on the free-rider effect.

Free-Rider Effect The free-rider effect captures the effect of greater investment on the employer's payoff through the employee's ideation. Recall from Lemma 1 that greater investment motivates the employer to ideate more: $\frac{\mathrm{d}a'}{\mathrm{d}I} > 0$. An increase in the employer's ideation demotivates the employee since he can simply free-ride off her effort to obtain his private benefit (Proposition 1): $\frac{\mathrm{d}e'}{\mathrm{d}a'} < 0$. Thus, greater investment reduces the employer's payoff by reducing overall innovation through the employee's ideation: $\frac{\partial V}{\partial e'} > 0$. On the whole, then, the sign of the free-rider effect is negative.

Lemma 3 (Equilibrium in the main setting)**.** *The equilibrium level of investment in the main setting is the 'I' that solves Equation (14) and is denoted by I^*. Substituting I^* into a' yields the employer's equilibrium ideation, denoted as a^*. Finally, substituting a^* into e' gives the employee's equilibrium ideation, denoted as e^*. The equilibrium is characterized by the triplet $\{I^*, e^*, a^*\}$.*

Proof. Follows from Equations (3), (7), and (14). ■

The closed-form expressions for the equilibrium quantities are in the appendix.

5.3 Comparing Investment Across Settings

There is this idea of starving your way to success...
— Julie Hanna*, Chairman of the Board, Kiva.org*

Proposition 3 (Underinvestment)**.** *The investment level in the main setting, I^*, is lesser than in the benchmark setting, I_{BM}.*

Proof. Since the direct cost of investment, $G(I)$, is the same across the benchmark and main settings, the comparison of the investment levels across the main and benchmark settings purely depends on the implementation and free-rider effects. In what follows, I discuss how these effects *individually* affect the comparison of I^* and I_{BM}.

1. **Implementation Effect**: Computing the implementation effect in the main setting gives:

$$\text{Main implementation effect} = (e' + (1 - e')a')X. \tag{15}$$

   Since $(e' + (1 - e')a') < 1$, the implementation effect in the main setting is lesser than the implementation effect in the benchmark, which is X (Equation (12)). Thus, the implementation effect implies $I^* < I_{BM}$.

   The logic for why the implementation effect is relatively smaller in the main setting is as follows. The implementation effect captures that greater investment is valuable since it increases the employer's payoff by increasing the likelihood of successfully implementing an idea. However, in the main setting, the existence of an idea is not a foregone conclusion, unlike in the benchmark. Therefore, the chance that an idea might not even be generated in the main setting reduces the value of sourcing investment relative to the benchmark.

2. **Free-Rider Effect**: The free-rider effect is negative in the main setting, while it is absent in the benchmark setting. Thus, the free-rider effect implies that $I^* < I_{BM}$.

Overall, the implementation and free-rider effects imply that the employer invests less in the main setting relative to the benchmark, completing the proof. ■

A lay observer would expect the employer's investment level to be set at I_{BM}. This is because I_{BM} is the NPV-maximizing investment level *provided* that an idea is generated. The main insight here is that underinvestment (relative to I_{BM}) spurs the employee to ideate and is, thus, payoff-increasing for the employer.

Proposition 3 has a flavor of 'money burning.' In the main setting, the employer destroys value in the implementation stage by underinvesting ($I^* < I_{BM}$). However, this money burning has a positive effect on the employee's ideation. The employee anticipates that since the employer's expected value from the implementation stage is smaller (due to the money burning), she has a smaller incentive to ideate and move on to the implementation stage. Consequently, free-riding is less profitable, and he must ideate more if he wants his private benefit.

Corollary 1 (Comparison of Expected Payoff)**.** *The employer's expected payoff is greater in the benchmark relative to the main setting.*

Proof. See appendix. ■

The broad intuition for this result is that the employer is already halfway through the innovation process in the benchmark (since she has an idea on hand) while she is at the start of the innovation process in the main setting. Thus, her expected value from the innovation is greater in the benchmark.

5.4 Discussion

Holding all-else-equal, an increase in the probability of successful ideation increases the employer's payoff, and Proposition 2 shows that greater investment can increase the probability of successful ideation. Thus, one might expect that adding ideation to a no-ideation benchmark leads to overinvestment (at least in some settings). However, Proposition 3 shows that adding ideation to a no-ideation benchmark always leads to underinvestment. What explains this puzzle?

The crux of the solution to this puzzle is that all-else is *not* held equal in the employer's payoff when the probability of successful ideation increases. An increase in the probability of successful ideation arises via two channels – an increase in the employee's and/or an increase in the employer's ideation levels. As expected, an increase in the employee's ideation (the first channel) increases the employer's payoff. However, an increase in the employer's ideation (the second channel) does not affect the employer's payoff. This is an application of the envelope theorem. Since the employer chooses her ideation optimally, any increase in her ideation increases the expected value of the innovation by the same amount as the cost of ideating, leaving the employer's payoff unchanged, as shown by the third term in Equation (14).

Thus, even though greater investment can increase the probability of successful ideation (Proposition 2), the employer only cares about how investment affects the probability of successful ideation through the employee's ideation; she does not care about how investment affects her own future ideation. Since greater investment reduces the employee's ideation due to free-riding (Proposition 1), the employer always underinvests in the main setting relative to the benchmark.

6 Employee Characteristics and Investment

Is investment a substitute or complement to employee creativity? How does the employee's implicit motivation affect investment? I explore these questions in this section. I use the answers to these questions to understand how employee characteristics affect the magnitude of underinvestment in firms.

Proposition 4 (Employee Characteristics and Investment)**.** *The equilibrium level of investment, I^*:*

1. *increases in the employee's creativity. Formally,*

$$\frac{\mathrm{d}I^*}{\mathrm{d}l} < 0. \tag{16}$$

2. *increases in the employee's private benefit, ψ. Formally,*

$$\frac{\mathrm{d}I^*}{\mathrm{d}\psi} > 0. \tag{17}$$

Proof. See appendix. ■

Part 1 of Proposition 4. An increase in l affects I^* through the implementation and free-rider effects.

Consider first the effect of an increase in l on the implementation effect. An increase in l implies that the employee is less likely to succeed at ideating; thus, the likelihood that an investment will be needed to implement the idea is lower, prompting the employer to reduce I^*.

On the other hand, an increase in l implies that the employee's ideation effort is primarily determined by his cost of ideation. Thus, any free-riding prompted by an increase in the investment is small and has less impact on the employer's payoff, prompting the employer to increase I^*. Formally, a larger l makes the free-rider effect less negative.

The effect of an increase in l on the implementation effect dominates, leading to a negative relation between l and I^*.

Part 2 of Proposition 4. As with part 1, an increase in ψ affects I^* through the implementation and free-rider effects.

As ψ increases, the employee increases his ideation. Intuitively, moving on to the implementation phase is more valuable to the employee, prompting him to engage in greater ideation. Since the employee is likely to generate an idea, the employer responds by setting aside more investment for implementation. That is, an increase in ψ increases the positive implementation effect.

However, since the employee's ideation is larger for a greater ψ, any free-riding generated by an increase in investment has a consequential impact on the employer's payoff. Formally, an increase in ψ makes the negative free-rider effect even more negative.

The effect of a greater ψ on the implementation effect dominates, leading to a positive relation between ψ and I^*.

Corollary 2 (Employee Characteristics and Underinvestment)**.** *Underinvestment is smaller when:*

1. *the employee is more creative (l is smaller).*
2. *the employee has a stronger implicit incentive (ψ is larger).*

Observe that $(I_{BM} - I^*)$ captures the magnitude of underinvestment, and I_{BM} does not depend on l or ψ (Equation 12). Then, Corollary 2 follows from Proposition 4.

Corollary 3 (Investment, Creativity, and 'Underperformance')**.** *As sourcing investment gets cheaper for the employer, a more creative employee is less likely to generate an idea.*

The idea underlying Corollary 3 is as follows. Greater employee creativity obviously has a positive *direct* effect on the probability that the employee generates an idea:

$$\frac{\partial e^*}{\partial l} < 0. \tag{18}$$

After all, a more creative employee faces a smaller cost of ideating.

However, greater creativity increases the employer's investment (Proposition 4). The employer realizes that the creative employee will likely generate an idea, and she responds by setting aside greater investment funds to implement the idea. This increase in investment increases the employee's incentive to free-ride off the employer's ideation (Lemma 1 and Proposition 1). Greater investment increases the employee's expected value from generating an idea, prompting her to increase her ideation. In turn, the employee decreases his ideation because he rationalizes that he will likely obtain his private benefit even if he does not generate an idea. Thus, greater creativity *indirectly* decreases the probability that the employee generates an idea:

$$\frac{\partial e^*}{\partial a^*} \cdot \frac{\mathrm{d}a^*}{\mathrm{d}I^*} \cdot \frac{\mathrm{d}I^*}{\mathrm{d}l} > 0. \tag{19}$$

As the cost of sourcing the investment gets cheaper – g gets smaller – the indirect effect gets large. Thus, the direct positive effect of creativity on idea generation is reduced by the indirect negative effect.[8]

An alternate interpretation is as follows: relative to the case where investment is exogenously fixed, a more creative employee 'underperforms' (is less likely to generate an idea), and a less creative employee 'overperforms' when investment is endogenously determined. The idea is that when investment is exogenously fixed, the indirect negative effect of employee creativity on idea generation through the employer's investment is held constant. Thus, an increase (or decrease) in employee creativity has the expected effect on the employee's idea generation. On the other hand, when the investment is endogenously chosen, an increase in employee cre-

[8]It is easy to check that the indirect effect never *dominates* the direct effect.

ativity *directly* has the expected effect on the employee's idea generation; however, greater creativity *indirectly* generates a countervailing effect on the employee's idea generation through the employer's equilibrium investment.

7 Incentive Contracting

For simplicity, the main analysis has focused on an incomplete contracting approach. One might wonder, however, if the results change when the employer can provide the employee with an incentive contract. In this subsection, I show that the main result in Proposition 3 is robust to the addition of an incentive contract.

The change I introduce to the model is that when the employee succeeds at generating an idea, the employer pays him a bonus W.[9] The employer sets W at Date 0. Thus, the employer's payoff, V, (in Equation (1)) can be amended to V^W to account for the cost of paying out the bonus:

$$V^W := e(IX \underbrace{-W}_{\text{Bonus}}) - (1-e)\left(aIX - \frac{ca^2}{2}\right) - G(I). \tag{20}$$

Analogously, the employee's payoff, Υ, (in Equation (2)) can be amended to Υ^W:

$$\Upsilon^W := e \underbrace{W}_{\text{Bonus}} + (e + (1-e)a)\psi - \frac{le^2}{2}. \tag{21}$$

Note that the employer's optimal ideation (for an arbitrary level of investment), a', is not influenced by the introduction of the bonus, W, since the employer ideates only *after* the bonus is paid.

Anticipating a', the employee solves the following:

$$\max_e \ \Upsilon^W(a') = eW + (e + (1-e)a')\psi - \frac{le^2}{2}. \tag{22}$$

I denote the arg max to this problem as $\hat{e}^W$; obviously, $\hat{e}^W$ depends on the bonus, W. Anticipating $\hat{e}^W$, the employer's problem with respect to the optimal contract is:

$$\max_W \ V^W(W, \hat{e}^W(W)). \tag{23}$$

[9] It is well known from standard results in agency theory that it is suboptimal to reward a risk-neutral employee for failure.

The solution to her problem is characterized by the first-order condition:

$$0 = \frac{\partial V^W}{\partial W} + \frac{\partial V}{\partial \hat{e}^W} \cdot \frac{\mathrm{d}\hat{e}^W}{\mathrm{d}W}. \tag{24}$$

The first term on the right is the direct cost of providing a larger bonus – the employer incurs a larger cost of compensation. The second term on the right is the incentive effect of a higher bonus – a larger bonus motivates the employee to ideate. I denote the W that solves the first-order condition as W'. Observe that W' is a function of the investment, I. Plugging in W' into $\hat{e}^W$ yields $\bar{e}^W$.

In choosing its investment, the employer's first-order condition (with a slight abuse of notation) is:

$$\begin{aligned} 0 = &\frac{\partial V^W}{\partial I} + \frac{\partial V^W}{\partial \bar{e}^W} \cdot \frac{\mathrm{d}\bar{e}^W}{\mathrm{d}a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I} + \frac{\partial V^W}{\partial a'} \cdot \frac{\mathrm{d}a'}{\mathrm{d}I} - \frac{\mathrm{d}G}{\mathrm{d}k} \\ &+ \left(\frac{\partial V^W}{\partial W'} + \frac{\partial V^W}{\partial \bar{e}^W} \cdot \frac{\mathrm{d}\bar{e}^W}{\mathrm{d}W'}\right) \cdot \frac{\mathrm{d}W'}{\mathrm{d}I}. \end{aligned} \tag{25}$$

The first line is identical to Equation (14). The difference in this first-order condition is the second line – the second line captures the effect of greater investment on the employer's payoff through the optimal contract. However, as Equation (24) shows, the second line is zero; this is an application of the envelope theorem.

Thus, the employer's optimal investment in the setting with an incentive contract is characterized by the same tradeoffs as in Lemma 3. Then, Proposition 3 applies as is, and it is without loss of generality and intuition to focus on the setting with an incomplete contracting approach.

8 Conclusion

In many settings, firms will seemingly underinvest in ideas relative to what is predicted by the NPV rule. This self-sabotaging is puzzling. If a firm knows that the likelihood of taking an idea to the market as an innovation (subject to a cost of investment) is maximized at X dollars, why would it invest less than X? I offer a solution to this puzzle.

The idea underlying my main result is that of free-riding. An employee (he) and employer (she) sequentially engage in ideation. If an idea is generated, the employee obtains a private benefit, and the employer generates a profit if the idea is successfully implemented. Thus, the employee and the employer both benefit when an idea is generated. However, the fact that both the employee and employer benefit from the generation of an idea implies that the employee free-rides off the employer's ideation. Intuitively, the employee anticipates that even if he undersupplies ideation effort, the employer will bail him out by ideating because she wishes to generate a profit.

The employer *ex-ante* chooses an investment level. The investment level affects the likelihood that the implementation of an idea is successful. In a benchmark setting where no ideation is required, the investment level is simply chosen to maximize the probability of successfully implementing the idea subject to a cost of sourcing the investment.

When ideation is added to the benchmark setting, an increase in investment plays an additional role over increasing the probability that an idea is successfully implemented. Specifically, since greater investment increases the employer's value from generating an idea (by increasing the likelihood that the idea will be successfully implemented), the employer's incentive to ideate increases. The employee anticipates this increase in the employer's ideation and responds by free-riding more. Thus, the fundamental tradeoff faced by the employer while choosing the investment level in the main setting is this: greater investment increases the probability of successfully implementing an idea but decreases the probability that the employee generates an idea.

Since greater investment generates an additional *cost* in the main setting over the benchmark setting (the employee's free-riding), the employer *underinvests* in the main setting relative to the benchmark. Thus, when viewed from the lens of the benchmark, underinvestment is puzzling. However, when ideation is considered along with implementation, underfunding is an equilibrium outcome.

Implications The main implications of the model are as follows:

1. Greater investment reduces employees' incentives to ideate (Proposition 1).
2. In settings with greater R&D funding, ideas will be more likely generated at higher levels of the organizational hierarchy (Lemma 1 and Proposition 1).
3. Investment and idea generation within a firm have an ambiguous relationship (Proposition 2).

Depending on how creative employees are, greater investment may increase or decrease the probability that an idea is generated.

4. Underinvestment in firms in the context of innovation is not an aberration (Proposition 3).

   A naïve application of the NPV rule would have firms choose an investment level that maximizes the likelihood of implementing an idea subject to a cost of sourcing the investment. However, the implicit assumption is that firms have an idea on hand. When the innovation process is considered as a combination of ideation and implementation, the equilibrium outcome is underinvestment.

5. Underinvestment is greater in settings with more creative employees or employees with stronger implicit incentives (Proposition 4).

6. Employee creativity has a smaller (but nevertheless positive) effect on an employee's idea generation when investment is cheap to source (Corollary 3).

Extensions The model has been kept simple to cleanly highlight the main mechanism. Now, I briefly discuss some extensions to the model and what role these extensions might play on the main result of underinvestment.

- In the current model, project implementation purely depends on the employer's investment. However, in reality, project implementation will also depend on the employee's effort. One might expect project implementation to succeed with some probability $\mathbb{P}(e_2, I)$ where e_2 is the employee's effort in the implementation stage.

  Such an extension is straightforward to execute and leaves the main insight of the model unchanged. The reasoning is as follows. In both the benchmark and main settings, the implementation effect will now also include how greater investment affects the employer's payoff through the employee's implementation effort: $\frac{\mathrm{d}\mathbb{P}(e_2,I)}{\mathrm{d}e_2} \cdot \frac{\mathrm{d}e_2}{\mathrm{d}I}$.[10] However, the key difference between the main and benchmark settings is the free-rider effect, which is unaffected.

- Another extension might consider making the employee's private benefit a function of the investment: $\frac{\mathrm{d}\psi}{\mathrm{d}I} > 0$. For instance, an employee might enjoy

[10]The sign of this derivative is ambiguous and depends on whether the employee's effort in the implementation stage and investment are complements or substitutes.

more perks with more investment funds. Such an extension can be easily executed and adds some nuance to the model, but the main insight presented in this paper obtains.

The difference introduced by this extension will be that greater investment in the main setting will now also motivate the employee to ideate by increasing the magnitude of his private benefit: $\frac{\partial e'}{\partial \psi} \cdot \frac{\mathrm{d}\psi}{\mathrm{d}I} > 0$. Barring large values of $\frac{\mathrm{d}\psi}{\mathrm{d}I}$, the free-rider effect will dominate, leaving the insight of the model presented here unchanged. The model abstracts from this extension because the idea that leaving 'rents' induces effort (in this setting, ideation) has already been explored before in Arya et al. (2000).

- A final extension can consider adding more generality to the model. I have used functional forms for the costs and probabilities of successful ideation or implementation in the current model. A more general approach would do away with these functional forms. It is easy to execute this extension, leaving the model's main result unchanged. The functional forms are used only to facilitate the computation of the comparative statics in Proposition 4.

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Appendix

Closed-form Expressions for Equilibrium Quantities

Solving Equations (3) and (7) gives:

$$a' = \frac{IX}{c}, \text{ and} \tag{A.1}$$

$$e' = \frac{\psi(c - IX)}{cl}. \tag{A.2}$$

Substituting a' and e' from Equations (A.1) and (A.2) into the employer's payoff V in Equation 1 yields the employer's objective function:

$$\max_I V(a', e') = \frac{\psi(c - IX)}{cl} IX + \left(1 - \frac{\psi(c - IX)}{cl}\right)\left(\frac{IX}{c} IX - \frac{c\left(\frac{IX}{c}\right)^2}{2}\right) - \frac{gI^2}{2}. \tag{A.3}$$

The first-order condition to her problem is:

$$\frac{IX^2\left(1 - \frac{\psi(c-IX)}{cl}\right)}{c} + \frac{\psi X(c - IX)}{cl} + \frac{\psi I^2 X^3}{2c^2 l} - \frac{\psi I X^2}{cl} - gI = 0. \tag{A.4}$$

The second-order condition to her problem is:

$$\frac{-c^2 gl + c(l - 3\psi)X^2 + 3\psi I X^3}{c^2 l} < 0. \tag{A.5}$$

Solving the first-order condition in Equation (A.4) for I gives two roots:

$$I_1 = \frac{c^2 gl + c(3\psi - l)X^2 - \sqrt{c^2((cgl + (3\psi - l)X^2)^2 - 6\psi^2 X^4)}}{3\psi X^3}, \text{ and} \tag{A.6}$$

$$I_2 = \frac{c^2 gl + c(3\psi - l)X^2 + \sqrt{c^2((cgl + (3\psi - l)X^2)^2 - 6\psi^2 X^4)}}{3\psi X^3}. \tag{A.7}$$

To rule out one of the roots as a contender for the equilibrium investment, I^*, I substitute both into the second-order condition identified in Equation A.5 and check which root satisfies it.

Substituting I_1 into the second-order condition gives:

$$-\frac{\sqrt{c^2((cgl+(3\psi-l)X^2)^2-6X^4\psi^2)}}{c^2 l}<0. \tag{A.8}$$

Thus, I_1 is a possible contender for I^* since it satisfies the second order condition. Substituting I_2 into the second-order condition gives:

$$\frac{\sqrt{c^2((cgl+(3\psi-l)X^2)^2-6X^4\psi^2)}}{c^2 l}\not<0. \tag{A.9}$$

Therefore, I_2 can be ruled out as a contender for I^* since it fails to satisfy the second-order condition (I_2 *minimizes* the employer's payoff).

Straightforward but tedious algebra shows that $I_1 \in (0,1)$ due to Assumption 1. Thus, it must hold that $I_1 = I^*$ since it is a maximum and interior. Substitution of I^* into Equations (A.1) and (A.2) gives a^* and e^*.

Proofs

Proof of Proposition 2.

Proof. Using a' and e' from Equations (A.1) and (A.2), the effect of greater investment on innovation $\iota(a',e') = e' + (1-e')a'$) is:

$$\frac{\mathrm{d}\iota(a',e')}{\mathrm{d}I}=\frac{X(c(l-2\psi)+2I\psi X)}{c^2 l}. \tag{A.10}$$

Setting this derivative to zero and solving for l gives:

$$\frac{\mathrm{d}\iota(a',e')}{\mathrm{d}I}=0 \implies l=\hat{l}:=\frac{2\psi(c-IX)}{c}. \tag{A.11}$$

Further,

$$\frac{\mathrm{d}(\text{Equation (A.10)})}{\mathrm{d}l}=\frac{2\psi X(c-IX)}{c^2l^2}>0, \tag{A.12}$$

due to Assumption 1.

Thus, it must hold that for $l>\hat{l}$, $\frac{\mathrm{d}\iota(a',e')}{\mathrm{d}I}>0$, and for $l<\hat{l}$, $\frac{\mathrm{d}\iota(a',e')}{\mathrm{d}I}<0$. ∎

Proof of Corollary 1.

Proof. This result can be established in two steps. First, observe that holding investment at I^*, the employer's expected payoff is greater in the benchmark. Formally,

$$\begin{aligned} & V_{BM}(I^*) > V(I^*) \\ \implies & I^*X - G(I^*) > \ (e^* + (1-e^*)a^*)I^*X - (1-e^*)\frac{c(a^*)^2}{2} - G(I^*) \\ \implies & I^*X(1-(e^*+(1-e^*)a^*)) > -(1-e^*)\frac{c(a^*)^2}{2}, \end{aligned} \tag{A.13}$$

which is true because the left hand is positive and the right hand is negative.

Second, the employer optimally adjusts her investment upward from I^* to I_{BM} in the benchmark (Proposition 3), further increasing her expected payoff in the benchmark. Formally,

$$V_{BM}(I_{BM}) > V_{BM}(I^*) > V(I^*). \tag{A.14}$$

This establishes the result that the employer's expected payoff is greater in the benchmark. ■

Proof of Proposition 4.

Proof. Part 1 establishes a negative relationship between the employee's cost coefficient of ideation effort, l, and the equilibrium investment level, I^*. To see this result, I differentiate I^* with respect to l and study the sign of the derivative. Using the closed-form expression for I^* from Equation (A.6) and differentiating gives:

$$\frac{\mathrm{d}I^*}{\mathrm{d}l} = \frac{c\left(-\frac{\left(cg-X^2\right)\left(X^2(3\psi-l)+cgl\right)}{\sqrt{\left(X^2(3\psi-l)+cgl\right)^2-6\psi^2X^4}} + cg - X^2\right)}{3\psi X^3}. \tag{A.15}$$

Suppose that this derivative is positive, then testing $\frac{\mathrm{d}I^*}{\mathrm{d}l} > 0$ gives:

$$\begin{aligned} \implies & \sqrt{(cgl + X^2(3\psi - l))^2 - 6\psi^2X^4} > -\left(X^2(l-3\psi) - cgl\right) \\ \implies & \left(cgl - X^2(l-3\psi)\right)^2 - 6\psi^2X^4 > \left(cgl - X^2(l-3\psi)\right)^2 \\ \implies & -6\psi^2X^4 > 0, \end{aligned}$$

which is untrue. Then, it must hold that $\frac{\mathrm{d}I^*}{\mathrm{d}l} < 0$, proving part 1.

Part 2 establishes a positive relation between the employee's private benefit, ψ, and the investment level, I^*. The effect of a larger private benefit on the equilibrium investment is given by:

$$\frac{\mathrm{d}I^*}{\mathrm{d}\psi} = \frac{cl\left(X^2 - cg\right)\left(\sqrt{\left(cgl + X^2(3\psi - l)\right)^2 - 6\psi^2 X^4} - cgl + X^2(l - 3\psi)\right)}{3\psi^2 X^3 \sqrt{\left(cgl + X^2(3\psi - l)\right)^2 - 6\psi^2 X^4}}. \quad \text{(A.16)}$$

Assume that this derivative is negative. Then, testing $\frac{\mathrm{d}I^*}{\mathrm{d}\psi} < 0$ gives:

$$\begin{aligned} & \sqrt{\left(cgl + X^2(3\psi - l)\right)^2 - 6\psi^2 X^4} - cgl + X^2(l - 3\psi) > 0 \\ \Longrightarrow\ & \left(cgl - X^2(l - 3\psi)\right)^2 - 6\psi^2 X^4 > \left(cgl - X^2(l - 3\psi)\right)^2 \\ \Longrightarrow\ & -6\psi^2 X^4 > 0, \end{aligned}$$

which is untrue. Then, it holds that $\frac{\mathrm{d}I^*}{\mathrm{d}\psi} > 0$, proving part 2. ■